PAPER ID-411939

Subject Code: KAS103T

Roll No:

BTECH
(SEM I) THEORY EXAMINATION 2021-22
ENGINEERING MATHEMATICS-I

Time: 3 Hours | ***Total Marks: 100***

Notes:
- *Attempt all Sections and Assume any missing data.*
- *Appropriate marks are allotted to each question, answer accordingly.*

SECTION-A	Attempt **All** of the following Questions in brief Marks**(10X2=20)**	
Q1(a)	If the matrix A= $\begin{bmatrix} -1 & 2 & 3 \\ 0 & 3 & 5 \\ 0 & 0 & -2 \end{bmatrix}$, then find the eigen value of $A^3+5A+8I$.	1
Q1(b)	Reduce the matrix $\begin{bmatrix} 1 & 1 & 1 \\ 3 & 1 & 1 \end{bmatrix}$ into normal form and find its rank.	1
Q1(c)	Find the envelope of the family of straight line $y = mx + \frac{a}{m}$, where m is a parameter.	2
Q1(d)	Can mean value theorem be applied to $f(x) = \tan x$ in $[0, \pi]$.	2
Q1(e)	State Euler's Theorem on homogeneous function.	3
Q1(f)	Find the critical points of the function $f(x, y) = x^3 + y^3 - 3axy$.	3
Q1(g)	Find the area bounded by curve $y^2 = x \text{ and } x^2 = y$.	4
Q1(h)	Find the value of $\int_0^1 \int_0^x \int_0^{x+y} dx\, dy\, dz$.	4
Q1(i)	Find a unit normal vector to the surface $z^2 = x^2 + y^2$ at the point $(1, 0, -1)$.	5
Q1(j)	State Stoke's Theorem.	5

SECTION-B	Attempt **ANY THREE** of the following Questions Marks**(3X10=30)**	
Q2(a)	Find the characteristic equation of the matrix A= $\begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix}$, compute A^{-1} and prove that $A^8 - 5A^7 + 7A^6 - 3A^5 + A^4 - 5A^3 + 8A^2 - 2A + I = \begin{bmatrix} 8 & 5 & 5 \\ 0 & 3 & 0 \\ 5 & 5 & 8 \end{bmatrix}$.	1
Q2(b)	State Rolle's theorem and verify Rolle's theorem for the function $f(x) = \frac{\sin x}{e^x}$ in $[0, \pi]$.	2
Q2(c)	If u, v and w are the roots of $(\lambda - x)^3 + (\lambda - y)^3 + (\lambda - z)^3 = 0$, cubic in λ, find $\frac{\partial(u,v,w)}{\partial(x,y,z)}$.	3
Q2(d)	Find the volume bounded by the cylinder $x^2 + y^2 = 4$ and the plane $y + z = 4$ and $z = 0$.	4
Q2(e)	Apply Green's theorem to evaluate $\int_C [(2x^2 - y^2)dx + (x^2 + y^2)dy]$,where C is the boundary of the area enclosed by the x-axis and the upper half of the circle $x^2 + y^2 = a^2$.	5

SECTION-C	Attempt **ANY ONE** following Question Marks **(1X10=10)**	
Q3(a)	Find the value of k for which the system of eqations $(3k-8)x + 3y + 3z = 0$, $3x + (3k-8)y + 3z = 0$, $3x + 3y + (3k-8)z = 0$ has a non-trivial solution.	1
Q3(b)	Find the eigen values and eigen vectors of matrix A= $\begin{bmatrix} 2 & 1 & 1 \\ 2 & 3 & 2 \\ 3 & 3 & 4 \end{bmatrix}$.	1

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SECTION-C	Attempt **ANY ONE** following Question	Marks **(1X10=10)**
Q4(a)	If $f(x) = \frac{x}{1+e^{\frac{1}{x}}}$; $x \neq 0$ and $f(0) = 0$, then show that the function is continuous but not differentiable at $x = 0$.	2
Q4(b)	If $y = \left(x + \sqrt{1+x^2}\right)^m$, find $y_n(0)$.	2

SECTION-C	Attempt **ANY ONE** following Question	Marks **(1X10=10)**
Q5(a)	Expand x^y in powers of $(x-1)$ and $(y-1)$ up to the third-degree terms and hence evaluate $(1.1)^{1.02}$.	3
Q5(b)	A rectangular box which is open at the top having capacity 32c.c. Find the dimension of the box such that the least material is required for its constructions.	3

SECTION-C	Attempt **ANY ONE** following Question	Marks **(1X10=10)**
Q6(a)	Change the order of integration in $I = \int_0^1 \int_{x^2}^{2-x} xy\,dy\,dx$ and hence evaluate the same.	4
Q6(b)	Find the position of the C.G. of a semicircular lamina of radius, a if its density varies as the square of the distance from the diameter.	4

SECTION-C	Attempt **ANY ONE** following Question	Marks **(1X10=10)**
Q7(a)	Find the directional derivative of $\nabla(\nabla f)$ at the point $(1,-2,1)$ in the direction of the normal to the surface $xy^2z = 3x + z^2$ where $f = 2x^3y^2z^4$.	5
Q7(b)	Find the constants a, b, c so that $\vec{F} = (x + 2y + az)\hat{\imath} + (bx - 3y - z)\hat{\jmath} + (4x + cy + 2z)\hat{k}$ is irrotational and hence find function $\emptyset$ such that $\vec{F} = \nabla\emptyset$.	5